Problem 1.12

Constructing a vector to a point
Consider two points located at $\mathbf{r}_1$ and $\mathbf{r}_2$, separated by distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Find a vector $\mathbf{A}$ from the origin to a point on the line between $\mathbf{r}_1$ and $\mathbf{r}_2$ at distance xr from the point at $\mathbf{r}_1$ where x is some number.

Solution

The general equation for a line in three dimensions is

$$\mathbf{y} = \mathbf{m}x + \mathbf{b}.$$

Since we're given two points, we can determine the two unknowns, $\mathbf{m}$ and $\mathbf{b}$. When $x = 0$, we're at $\mathbf{r}_1$, and when $x = 1$, we're at $\mathbf{r}_2$. The two equations we can write from these conditions are the following.

$$\begin{aligned}\mathbf{r}_1 &= \mathbf{m} \cdot 0 + \mathbf{b} \\ \mathbf{r}_2 &= \mathbf{m} \cdot 1 + \mathbf{b}\end{aligned}$$

Solving this system with substitution, we obtain $\mathbf{m} = \mathbf{r}_2 - \mathbf{r}_1$ and $\mathbf{b} = \mathbf{r}_1$. Therefore, the equation of the line that goes through $\mathbf{r}_1$ and $\mathbf{r}_2$ is

$$\mathbf{y} = (\mathbf{r}_2 - \mathbf{r}_1)x + \mathbf{r}_1,$$

where $x \in [0, 1]$. $\mathbf{A}$ is the vector $\mathbf{y}$.

$$\mathbf{A} = (\mathbf{r}_2 - \mathbf{r}_1)x + \mathbf{r}_1$$